RISK POLICY AS A UTILITY FUNCTION

by John Schuyler

Contents

OVERVIEW

Risk attitude is an important dimension of decision policy. The purpose of this paper is to assist in determining risk policy as a utility function. The risk policy might be for an individual or for senior management of a corporation. For convenience, the appropriate perspective will be call *the decision maker*.

Risk policy expresses how the decision maker measures and feels about potential outcomes. We will focus on wealth creation as the goal. The most popular measure in capital investment analysis is *net present value* (*NPV* or *PV*).

Decision analysis (DA) is an approach and process for making decisions under uncertainty. DA applies to all manner of decisions, including multi-criteria decision making for individuals, governments, non-profits, and other entities with multiple or complex objectives.

Decision Policy

We can base a complete decision policy entirely on how the decision maker chooses to measure value. Policy is often decomposed into three preferences or attitudes:

- Objective(s)
  Creating wealth? Perhaps that plus several corporate social responsibility (CSR) areas? Providing high quality of life for an individual or for citizens of a community?
- Time preference
  Present value discounting is universal if timing matters.
- Risk attitude
  Guiding how to trade-off risk versus value

Business decision policy is easiest, because the mission is clear—at least, more clear. Companies who declare the “maximizing shareholder value” objective typically adopt *NPV* as their value measure.

With multiple objectives, crafting the decision policy is more challenging. One simplifying approach is to use monetary-equivalents for objectives not measured as money. These can be made as either *NPV*-adjustments or as cashflow adjustments. This is usually adequate, especially where wealth creation is the foremost objective.

The more-general approach is to craft a multi-objective value function. In operations research, we call this the *objective function* to be optimized. Though risk policy can be devised for any type of value measure, our discussion will stay with NPV maximization as the objective.

If the preference or culture is to be risk-averse, then a conservative risk policy is appropriate. The risk policy method that will be discussed is commonly—though not universally—recognized as best practice.

Expected Value

The central calculation in DA is *expected value* (*EV*), synonymous with *mean*. *EV* is simply the probability-weighted outcome:[1]

$$EV = \sum_{i=1}^{n} p_i x_i$$

where x_i is outcome value i

p_i is the probability of x_i

n is the number of possible outcomes.

When *NPV* is the measure, the *EV* is called ***expected monetary value*** (***EMV***).

On occasion, you may see synonyms for *EMV*, including:

mean *NPV*

expected *NPV*

E(*NPV*)

EV *NPV*

Many companies adopt an EMV decision policy:
Choose the alternative with the highest *EMV*.

In a portfolio context, EMV companies optimize their decision variables to maximize *EMV* for the portfolio.

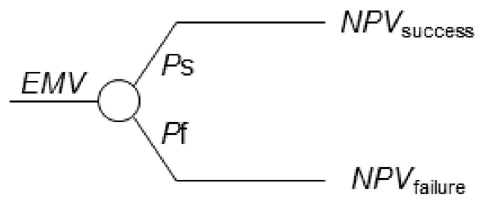
This *EMV* decision policy assumes the company is risk-neutral and there is no capital constraint. Later discussion includes popular ranking metrics for corporate planning with a capital constraint.

The EMV decision policy assumes complete lack of emotion about the risk: Complete objectivity about money or monetary-equivalents. The width or shape of the *NPV* outcome distribution does not matter, only its *EV*, the *EMV*.

[1] Greek capital sigma, Σ, is mathematics notation for a summation. *EV* is the sum of products: probabilities times values.

Simple Chance Node

Here is an example binary risk (or chance) event for project outcomes as would be represented in a decision tree:



Binary chance event. Ps and Pf are the probabilities of success and failure, respectively.

Projects typically have a wide continuum of possible outcomes, instead of just success and failure. The method, ahead, is fully and easily extendable to a continuum of possible outcomes. Keeping examples simple, with only two discrete outcomes, will make it easier for you to judge value and risk.

Capital Constraint

It always comes up. In discussion of the EMV decision rule, the matter of a capital constraint usually surfaces. EV calculations still apply, but decision policy becomes less clear.

Companies who are capital constrained often rank candidate investments with *discounted return on investment*:

$$DROI = \frac{EMV}{\text{EV PV Investment}}$$

This is a simple criterion that calculates *EMV* added (numerator) per unit of a capital constraint (in the denominator). Projects are approved, in rank sequence, until the capital budget is fully allocated.

Although there may be better projects, both lower-*EMV* or lower-*DROI* projects should be done also. The PV discount rate assumes money is available at that interest rate. Then, using either *EMV* or *DROI*, the optimal portfolio includes all projects with positive *EMV*.[2]

Other popular ranking metrics include:

IRR internal rate of return
A PV discount rate solution that makes $NPV = \$0$.
Sometimes, there are two *IRR* solutions.

PI profitability index
Same as *DROI* except that the investment is not discounted (which I prefer).

---

[2] If a company is risk-neutral and still turns down some $EMV > \$0$ projects, then perhaps they are using a PV discount rate that is too low.

These days, with ridiculous low interest rates, it is hard to assign a high number to cost of capital. The world is awash in money, with investors looking for yield. More often, the constraint is people or some other scarce resource. You can replace the *DROI* or *PI* denominator with units of whatever constraint keep you from higher *EMV*.

Decision Analysis

EV is central to decision analysis. The workhorse tools for *EV* calculations are decision trees and Monte Carlo simulation. They solve in very different ways, and each method has its advantages.

The Most Important Reason

Project, cashflow, and other models that use probabilities are called *stochastic models*. They may be also called *probabilistic models*. Solving the model effectively carries probability distribution inputs through the calculation. The resulting outcome, such as *NPV*, is a distribution. And for that distribution, we calculate its *EMV*.

A conventional model does not use probabilities. Model inputs are best single-value judgments. This is commonly called a *base case* model. An *NPV* is the typical primary outcome.

In a new venture analysis, the base case *NPV* was $1.20 million.[3] The *EMV* was $4.58 million. The *EMV* is the better number for appraising the project. The -$3.38 million *correction* to the base case value is called *stochastic variance* (*SV*).

The most important reason to use decision analysis is to get better values.

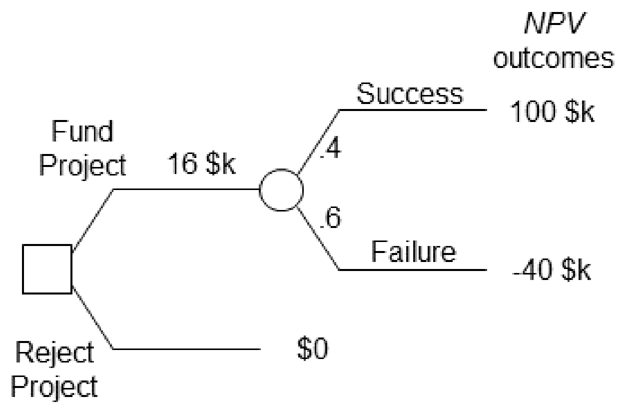
Large *SV*'s are typically caused by:

- Asymmetric outcomes. In the new venture case, it was the option to terminate the business if not producing positive cashflow after two years.
- Options and other complexities written into contracts
- Correlations among variables.
- Future decision points where resources can be redirected in the face of changing circumstances.

[3] Appendix 13A in my *Risk and Decision Analysis, 2nd Edition*, 2001, Project Management Institute. Stochastic Variance is the leading topic in Chapter 26, "Variance Analysis," in *Risk and Decision Analysis, 3.0 Edition*, 2016.

Simple Decision Tree

Let's extend the last figure into a little decision model:



Simple decision tree for a project investment. The *EMV* = $16k of funding the project is better than the $0 Reject alternative.

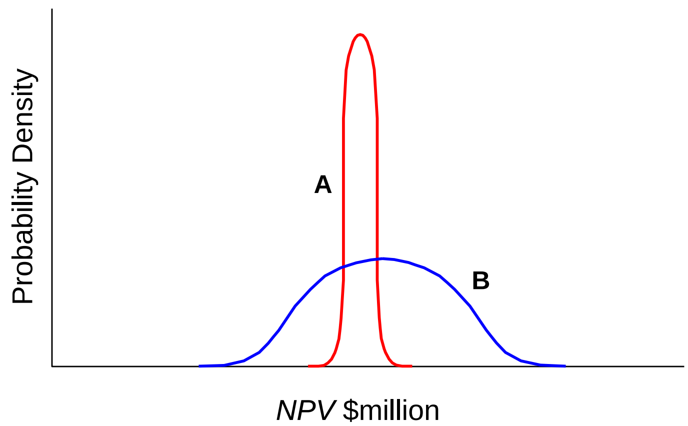
Simple risk experiments, called *lotteries*, is a good way to represent and experience the concepts.

In this simple tree, there is only the single risk event. It is not uncommon for decision trees to have hundreds or even thousands of branches. One advantage of Monte Carlo simulation is being able to directly use judgments about uncertainty expressed as continuous distributions.

Need for Risk Policy

Common advice: "To make more money, you need to take more risk."

Often, the highest *EMV* alternative has more risk, as illustrated.



Alternative A has less risk but a lower *EMV*. Alternative B has a higher *EMV* and more risk.

A risk-neutral decision maker does not care about the width of the distributions and would always choose the higher *EMV*.

A conservative person has a more difficult decision. A meaningful, quantitative risk policy will clearly identify the better alternative.

Risk Policy

Discussing decision policy can fill volumes.[4] This section presents highlights, with a focus on the risk policy component.

We will consider investing in capital investment projects or acquiring risky assets ("Buy" context). The analysis to divest or sell a risky project or asset is identical ("Sell" context).

To keep it simple, each project will be characterized by a "success" or superior outcome, represented by its *NPV* (*net present value*) and symbolized as NPV_S. Similarly, the counter "failure" outcome will be labeled NPV_F. In some cases, these outcomes may have the same sign.

The Most Important Metric

In buying or selling, the most important piece of information is the value of the service or asset. For you, that is your *CE*. And that value is almost always an estimate. This estimate may be the result of:

- Judgment, based upon experience and intuition
- A value equation (which might be *NPV* of a net cash flow forecast), with careful assessment of input variables
- Substantial relevant history of transactions of like or similar assets or projects
- Some combination of these methods

To be most useful to the decision maker, estimates should be reasonably precise and unbiased. Poor project and asset appraisals lead to these business performance issues:

- If your value estimates tend to be too low: You will seldom be able to acquire assets unless the seller also under-estimates the value or makes an assessment error.
- If your value estimates tend to be too high: You will most often overpay for acquisitions.
- If your estimates are, on average, unbiased but have substantial random errors: The projects that you approve and acquisitions

[4] UEP is an on-line companion to *Decision Analysis for Petroleum Exploration, 3+ Editions* (Newendorp and Schuyler) and *Risk and Decision Analysis in Projects, 3+ Editions* (Schuyler), and a planned *Decisions with Risk* (a decision maker's guide).

> that you are able to make will tend to be those where you happened to be optimistic.[5]

Some disciplines produce conservative value estimates. Financial statements and mineral property appraisals come to mind. Conservative estimates are a disservice to decision makers. Instead, I recommend that estimators and analysts attempt to make their assessments as objective and as precise as reasonably possible. The decision maker will also benefit with a characterizations of uncertainty, e.g., a distribution of *NPV* in addition to the *EMV*.

Still, there is often aversion to risk. Rather than producing biased, conservative estimates, a conservative risk attitude can be handed in a logical, consistent way by risk policy.

Expected Value Utility

While the EMV decision policy is appropriate for most daily decisions, risk aversion often is significant in decisions with large value outcomes and/or high risk of failure.

People and companies often override the EMV decision policy because of risk. They know they are risk-averse or risk-avoiding. They need a *risk policy* to guide in making trade-offs between risk and value.

Advisors often say, “It depends upon the investor’s risk attitude.”

Of course it does! But, that statement is of little help. Read on to learn how to implement a logical, consistent risk policy.

First, let’s define several terms that will be useful:

utility: a synonym for *value*, often used by economists, psychologists, and decision scientists.

expected utility (*EU*): EV of the outcome measured in utility units.

certain equivalent (*CE*): the value of *EU* translated into customary units. If value is measured in NPV$, then *CE* is also in NPV$.

We are all *EU* maximizers. The utility of an investment or project outcome depends upon different performance metrics. Each of us is unique in how we perceive the value of different objective measures, time value, and risk.

A *utility function* is a compact and consistent way to represent risk policy. This is needed for decisions where the magnitude and uncertainty of potential outcomes is important. Every person and organization has a utility function that can be deduced from experience and or elicited with simple experiments.

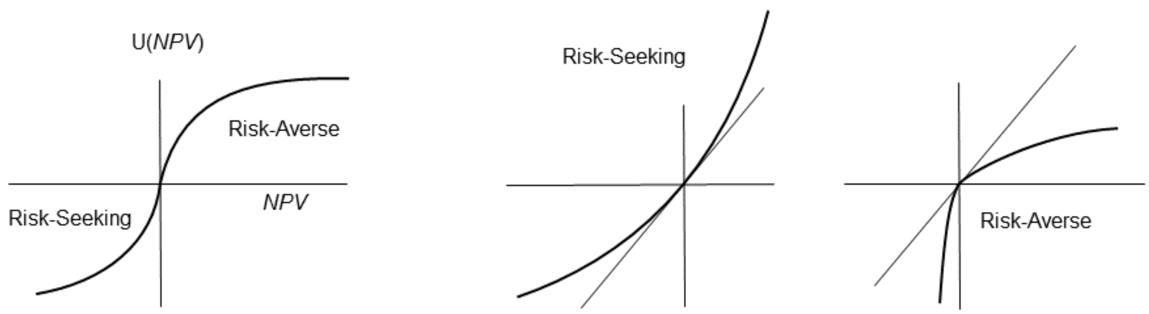
[5] These lead to *optimizer’s curse* and *winner’s curse* biases.

A *utility function* used for risk policy is simply a curve that translates an objective measure of value, such as *NPV*, into utility units.

Historical note: The earliest formal reference to utility theory is a 1738 article by Daniel Bernoulli. He solved the "St. Petersburg Paradox" proposed by his brother in 1713. John von Neumann and Oskar Morgenstern more-fully developed expected utility theory in in their 1947 landmark book, *Theory of Games and Economic Behavior*.

Utility Functions

Typically, risk preference is represented by a utility function. Here are several examples:



Left: Psychologists Daniel Kahneman and Amos Tversky documented people exhibiting inconsistent risk attitudes: People often demonstrate risk seeking behavior for losses and risk-avoiding behavior for gains.[6]

Center: People appear to be risk-seeking when gambling in casinos or buying lottery tickets. I prefer to think they experience value in the gaming experience, because these are certainly not economic decisions. A risk-seeking person's utility function accelerates upward.

Right: More appropriate is a consistently conservative risk attitude. These utility functions are "concave downward." Often, there is a sharp elbow at the origin.

You may have heard of "the law of marginal utility." As we get more of a good thing, the next unit is less valuable to us than the prior unit. $2 million is not twice as valuable that $1 million to the typical person. The utility function's slope diminishes moving to the right.

However, losses are amplified. Losing $100k is more than twice as bad as losing $50k. The curve is increasingly steep, moving leftward from $0.

[6] Daniel Kahneman's *Thinking, Fast and Slow*, 2011, discusses "prospect theory" and other cognitive biases. This is a magnificent, best-selling book. Kahneman shared in the 2001 Nobel Prize in economics. Tversky almost surely would have shared in the prize had he been still living.

The right two charts (figure above) include straight reference lines. A straight line utility function represents a risk-neutral decision maker.

Remarkably, a utility function represents a complete risk policy. We will see how the using provides a quantitative means to make consistent trade-offs between risk and value.

Exponential Utility Function

"Economic man" is consistent and rational. He optimizes his expected value utility. By convention, we call this ***expected utility*** and abbreviate ***EU***.

The exponential utility function shape is most popular. It is easy to calculate and work with.

Because of its highly desirable properties, this shape is most popular. I believe that using an exponential utility function is best practice, though not everyone agrees.

This is the only shape that has these highly desirable properties:

- The delta property. Adding $*X* to all outcomes increases the project certainty equivalent (*CE*) by $*X*. Recall, *CE* is the cash-equivalent of the risk.
- A risky project has the same value, whether buying or selling it. The value doesn't suddenly jump when the transaction closes.
- Calculating *value of information*, *value of control*, *value of flexibility*, and *value of robustness* alternatives are easier.
- The base or starting wealth does not matter, and we need to consider only the incremental changes.

The *exponential utility function* is the only shape that has these features. Therefore, it is widely accepted and applied by decision analysts. If we accept the exponential utility function shape, then defining the utility curve requires a single scaling parameter, the *risk tolerance* coefficient (*r*). The purpose of UER is to help you determine *r*.

"Exponential utility function" actually refers to a family of functions. They all effectively work the same, only with different utility units. The exponential utility function is scaled with a single parameter, the decision maker's ***risk tolerance coefficient*** (***r***).

To have a complete, consistent risk policy, all we need is *r*. This can be elicited by examining actual and/or hypothetical transactions.

We might be able to approximate *r* with a sufficient record of actual investment decisions—approvals as well as rejections. Using UEP allows you to consider a variety of hypothetical, simple project decisions.

There is a “family” of exponential utility functions. The utility scales may be different, though they perform identically in respect to decisions. I recommend this utility equation form:[7]

$$U(x) = r(1 - e^{-x/r})$$

where *U* is utility, the equation result, in risk neutral dollars (or whatever currency)

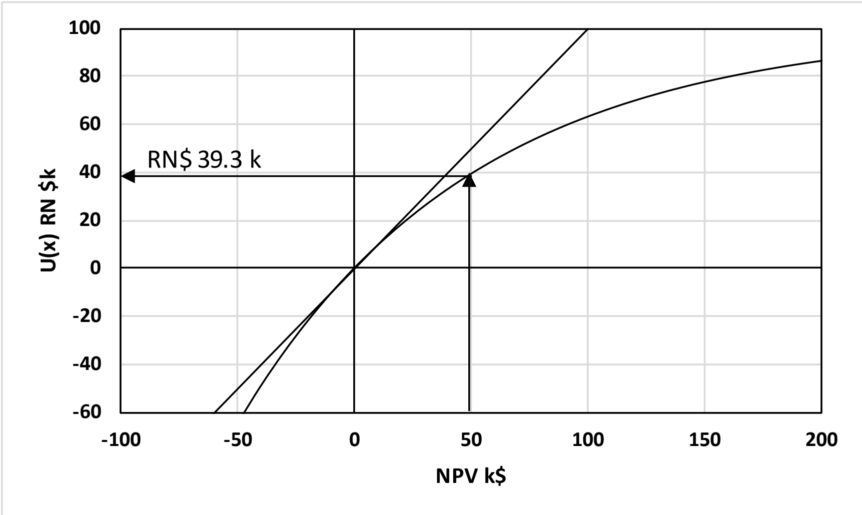
x is the outcome value, typically *NPV* dollars

r is the risk tolerance coefficient, in the same units as *x*

e is the natural log base, approximately 2.71828

I label the utility units *risk-neutral dollars* (RN$) (or RN other currency units). Many textbooks instead label utility in “utils” or “utiles.”

Here is an example plot of the exponential utility function:



Exponential utility function. The utility scale, *y*-axis, is measured in “risk-neutral dollars.” The risk tolerance coefficient (*r*) is $100k. As *NPV* increases, U(*NPV*) approaches *r*, that is, RN$100k. Conversion example: U($50k *NPV*) = RN$39.3k.

Note that as *NPV* increases, the curve continues upward though at a gradually decreasing slope. We always want more *NPV*, but the incremental utility of additional *NPV* units continues to diminish.

Publishing the chart is a way of distributing risk policy to the organization. However, it is nice to have an equation for utility, as a formula for convenience and accuracy. It is a good idea to check the calculation against the chart, as it is easy to make a calculation error.

[7] An e^x term is represented as EXP(x) in Excel and many other computer tools. This is the natural exponentiation function. This term is the basis for calling this an *exponential utility function.*

The *y*-axis is in “risk-neutral dollars.” These are not real dollars, though there is a modest connection: $100k RN$ is 100k times better than $1.

Note that near zero, the utility curve is nearly coincident with the 45° reference line. This means, with outcomes near zero, the decision maker is nearly risk-neutral. This includes most day-to-day decisions. This utility function is scaled by a risk tolerance coefficient (*r* or *RTC*) of $10 million.

The exponential utility function shape is always the same. It is a matter of scale, represented by *r* in the formula. You and Warren Buffet may have the same exponential shape, though Mr. Buffet has more zeros on the axes and you may have fewer zeros.

It is useful, though not necessary, to convert *EU* into *NPV*. This conversion would yield the *CE*. The same utility function is used, but this time we enter from *EU* at the *y*-axis. Fortunately, there is an equation for this inverse calculation:

$$CE = -r\ln(1 - EU / r)$$

where	*CE*	is the *certain equivalent*, the equation result
	EU	is expected utility
	r	is the risk tolerance coefficient
	ln	is the natural log function

Though not needed for decision making, determining the *CE* is often useful. The *CE* should be the indifference value or cost when buying or selling the risk. With small risk outcomes, the *CE* is close to *EMV*. The difference, *EMV* – CE = *risk premium* (*RP*). Thus, *RP* is the amount of *EMV* the decision maker is willing to give up to eliminate the risk.

How big is *r* ?

Here are two quick-and-dirty ways to get two initial, rough values for *r*.

Fraction of Net worth

Typically, *r* corresponds to about 1/5 of someone’s net worth or perhaps 1/5 of a company’s value. This rule of thumb is very personal and can vary by an order of magnitude or more depending upon the individual’s attitude toward risk.

+X, –X/2 Experiment

Here is an easy thought experiment to determine an approximate *r*. Let's suppose *X* = \$100. Your friend offers you a coin flip experiment.[8]

- If Heads, you receive *X* = \$100.
- If Tails, you pay him -*X* = \$50

Now, knowing about *EV*, you calculate your:

$$EMV = .5(\$100) + .5(-\$50) = \$25.$$

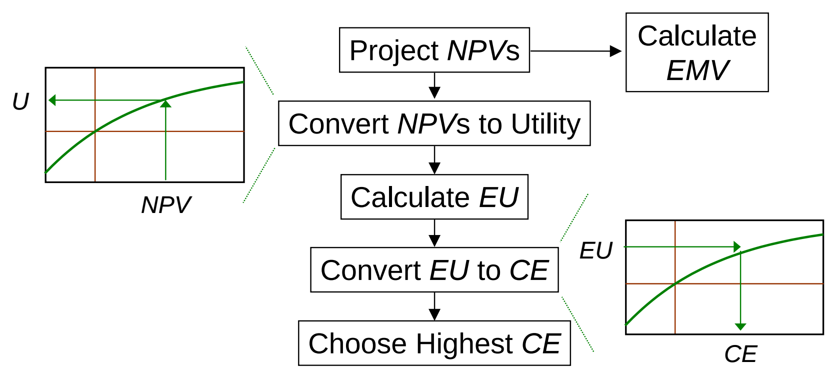
Would you accept the wager? Most people understanding *EMV* would. If you could repeat this experiment many times, over the long run your per-experiment average will be about \$25.

Now, consider the same experiment with step increases in *X*. \$200, \$400, …. At some point you would say, "No more!" The maximum *X* where you are just willing to accept the gamble is approximately your *r*. (The actual solution: *r* = 1.039*X*.)

Applying Risk Policy

While the transformation from actual money may seem awkward initially, this is needed because—being conservative—we are not linear in how we feel about money.

This diagram summarizes the process of evaluating alternatives:



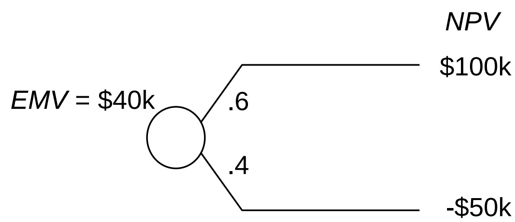
Instead of the EMV decision policy, the risk-averse person or organization uses the *expected utility decision policy*. That is, choose the alternative with the highest expected (value) utility (*EU*). Because utility in RN dollars isn't real money, it is helpful to covert *EU* back to real money, the *certain equivalent* (*CE*). Think of the *CE* as the cash-in-hand value of a risk situation or risky asset.

---

[8] If you, personally, dislike games of chance, please think of this in a business or investment context.

Example Utility Calculations

Let's consider a simple risk event:



The *EMV* is $40k, as shown.

Assume risk policy is an exponential utility function where the risk tolerance coefficient (*r*) is $100k.

In applying risk, policy, we first calculate the expected utility (*EU*).

Recall the equation:

$$U(x) = r(1 - e^{-x/r})$$

Let's do all the calculations in k units.

First, we convert the *NPV* outcomes into utility units:

$$\mathrm{U}(100) = 100(1 - e^{-100/100}) = \mathrm{RN\$63.21}$$
$$\mathrm{U}(-50) = 100(1 - e^{+50/100}) = -\mathrm{RN\$64.87}$$

Solving for *EU*:

$$EU = (.5)(63.21) + (.5)(-64.87) = \mathrm{RN\$11.98}$$

And now using *EU* to solving for *CE*:

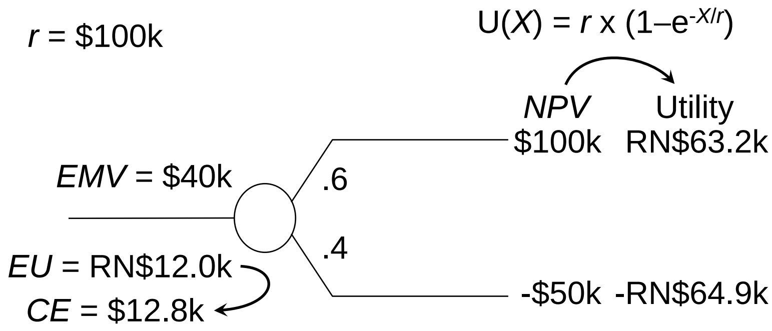
$$\begin{aligned} CE &= -r\ln(1 - EU/r) \\ &= -100\ln(1 - 11.98/100) \\ &= \$12.76 \end{aligned}$$

The risk premium is the value difference,

$$RP = EMV - CE = 40 - 12.76 = \$27.24$$

Thus, this decision maker is willing to give up $27.24k of *EMV* to realize $12.76k cash-in-hand.

This next figure summarizes the calculations:



$$EU = .6(63.2) + .4(-64.9) = RN\$12.0k$$

$$CE = -r \times \text{Ln}(1 - EU/r)$$

Utility calculations.

Applying risk policy in value calculations is very straightforward.

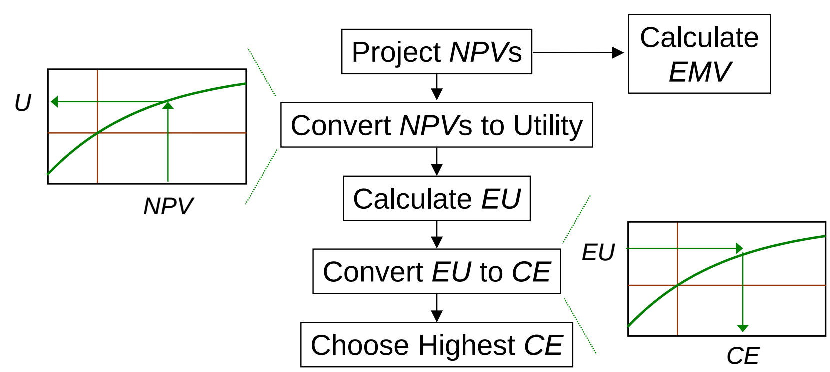
In decision tree software, such as PrecisionTree®, selecting to use utility, entering *r*, and viewing *EU* or *CE* values is all that is needed.

It is similarly easy with Monte Carlo simulation. With every *NPV* calculation, also calculate its utility value. Average that value (which may be a spreadsheet cell), and you have *EU*. Then convert to *CE*.

Using the Utility Curve

While the transformation from actual money may seem awkward initially, this is needed because—being conservative—we are not linear in how we feel about money.

This diagram summarizes the process of evaluating alternatives:



Instead of the EMV decision policy, the risk-averse person or organization uses the *expected utility decision policy*. That is, choose the alternative with the highest expected (value) utility (*EU*). Because utility in RN dollars isn't real money, it is helpful to covert *EU* back to real money, the *certain equivalent* (*CE*). Think of the *CE* as the cash-in-hand value of a risk situation or risky asset.

A convenient way to elicit *r* is to present simple, hypothetical decisions to consider. The simplest risk representation is a binary chance node, seen earlier, where the project outcome can be either Success or Failure. We characterize the risk with a probability of success (P_S). Thus, the Success outcome has a P_S chance of happening, and Failure has a $(1- P_S)$ chance of happening. The decision opportunity must have risk or there isn't a need for decision analysis.

Purpose of UEP

UEP is to help you or your organization craft a logical, consistent risk policy by eliciting risk preference. We assume the most-popular exponential utility function shape, which has highly desirable characteristics. Assuming this shape requires assessing only one scaling factor. This saves a tremendous amount of work in defining and applying the utility function.

Your, or your company's, preference for risk—captured in the utility function—should be reasonably stable. It should not change much unless there is a substantial change to your wealth or the company's value.

With practice, using UEP, you may become reasonably calibrated in making consistent risk versus value trade-offs in your decision making. The inferred *r*'s will converge with practice and feedback.

However, the point is not for you to become calibrated. Your emotions about risk may change from day-to-day. The *r* should not be mood dependent. You do, however, want decision policy to be consistent. Decision policy should be established in settled times. UEP will assist with the risk policy part.

Situations where someone might want to be calibrated about decision policy is when there isn't time for careful quantitative analysis, such as fast-paced, high impact situations. Examples include bidding in a live auction and decision-making in a crisis.

Simulations are good for training people to make good intuitive decisions in fast-paced, intense situations. Decisions under simulated stressful conditions can be scored against values determined separately with decision policy and situation modeling.

Three Question Formats

UEP presents you with random questions, scaled to your general maximum investment level. Provide a numeric answer using your experience and intuition. No calculations are required, though at some point you should do a few *EU* and *CE* calculations to prove to yourself that the equations work.

There are no wrong answers. Risk preference is a very personal attribute. Practice and feedback should reduce inconsistency in the returned risk tolerance coefficients.

Occasionally, you might accidentally provide an answer that would be risk-seeking. UEP will warn you not to do that.

Questions are generated in any of these three formats for a Buy (acquisition) perspective:

- What is Minimum acceptable probability of success (P_S) before you would approve this investment?
- What is the most you would pay (your *CE*) to acquire this risky project (or asset)?
- What is your optimal Share (your participation or ownership fraction) to acquire in this large, risky project?

The questions are worded a bit differently for a Sell context and for *CE* questions with outcomes of the same sign.

The question Graphics view adds a chance node graphic, such as the representation in a decision tree. A table presents supplemental information showing, as indexed by example answers.

We assume:

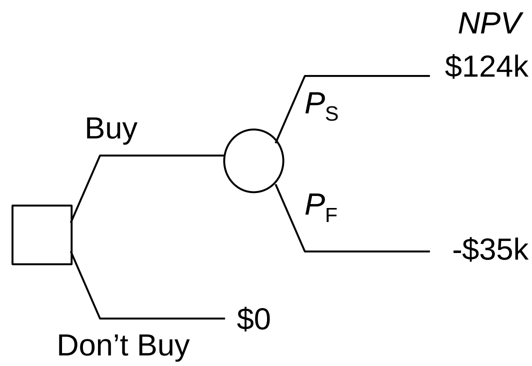
- These are individual projects (or assets or investments), reasonably independent of other projects in the portfolio.
- You are more constrained by lack of good projects than by lack of funds, insufficient staff, limited facility capacity, etc.
- These decisions important and infrequent. A corporate decision maker might face a major investment decision perhaps monthly, while an individual investor might make two important decisions per year.

All *NPV*, *EMV*, and *CE* amounts are in the same monetary units. The default label (including scale) is “$k”, which you can change. For the sample questions in this document, feel free to factor the outcome values to make them more appropriate to your situation. If you multiply the outcome *NPV*s by a factor, remember to change the returned *r* value by that factor.

Expected utility (*EU*) will be in risk neutral currency amounts, e.g., RN$k if the currency units are $k.

Write down your answers to the nine questions. The inferred *r* values will be revealed in charts at the end.

1. Probability of Success

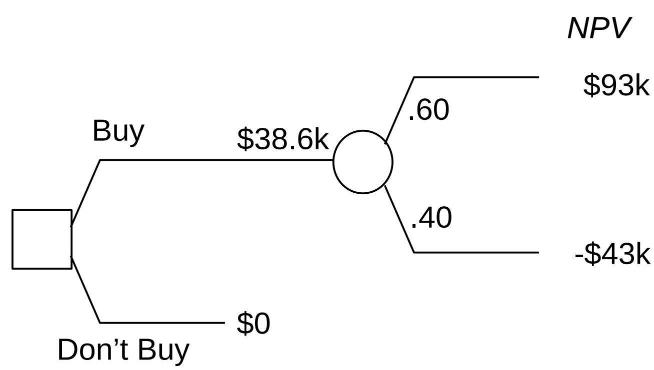


What is the *minimum* probability of success (P_S) that you would require to be *just willing to approve* this project investment?

For reference, the P_S that makes *EMV* = $0 is .220. You, as a risk-averse person would require a higher P_S.

Your risk-averse answer should be between .22 and 1.

2. Certain Equivalent



As shown in the tree, you have all the information to calculate the $38.6k *EMV*. What is your *certain equivalent*, *CE*, for this risk?

For the risk neutral person, *CE* = *EMV*.

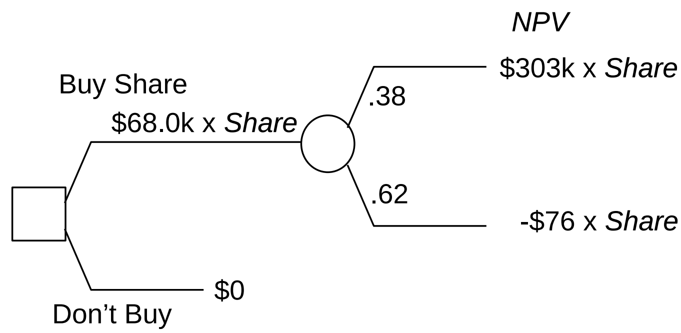
If you are risk-averse, the project or asset is worth less to you.

What is the maximum you would be willing to pay to acquire this risk?

This should equal the minimum amount you would sell it for if you already owned the risk.

Your risk-averse answer should be between -$43k and $38.6k.

3. Optimal Share



Consider investing in the above risky project. It is larger than you normally would consider, and fractional participation is offered. What is your optimal Share of this project?

It is an economic project. As such, you will always want a portion. This adds diversification to your portfolio.

You will always want part of project with *EMV* > $0. [9] This assumes you do not have a capital constraint: Any piece of a project with *EMV* > $0 will improve the *EMV* of the portfolio.

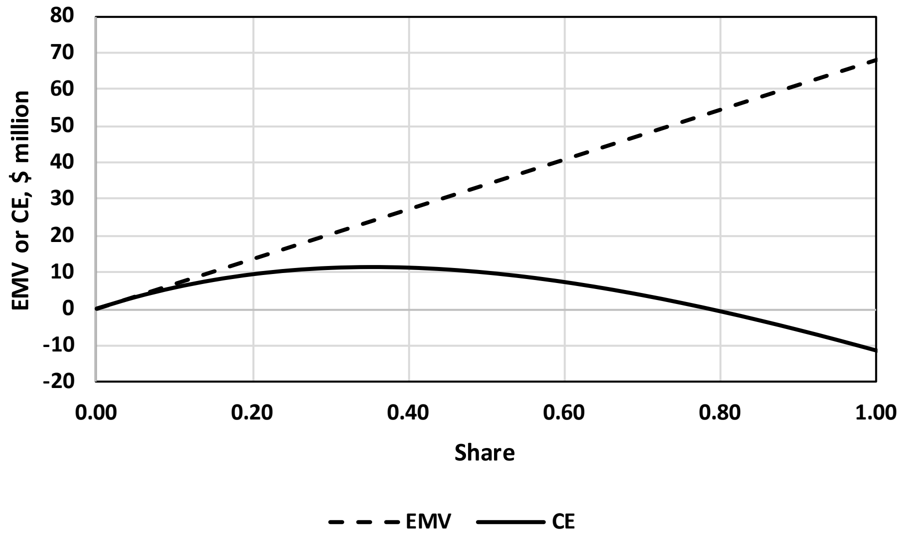
Without a capital constraint, the EMV decision policy says you want all or none of the project

The Share type question ask you to assess—intuitively—what *Share* you want in project. The optimal *Share* will be the participation fraction that provides you or your company with the greatest expected utility (*EU*) and greatest certain equivalent (*CE*).

An EMV decision maker would want all or none. A risk-averse decision maker may still want 1.00. For modest *r* values, more likely he or she will feel the optimum is a lesser fraction.

---

[9] We are assuming independent projects, so this is fully diversifiable risk. This is like free money! My statistics professor taught, "If the odds are in your favor, bet small and bet often."



Actual *EMV* line and hypothetical *CE* curve for this lottery. The shape of the CE curve depends an unspecified *r*. This is for illustration, only. **Try not to be influenced by the *CE* curve:** Your optimal Share might be very different from the approximately .35 optimum shown.

This chart may be of used in considering your optimal Share fractions:

	Share	Share	Share
Share	NPV Gain	NPV Loss	NPV
----------	----------	----------	----------
1.00	303.0	-76.0	68.0
0.90	272.7	-68.4	61.2
0.80	242.4	-60.8	54.4
0.70	212.1	-53.2	47.6
0.65	197.0	-49.4	44.2
0.60	181.8	-45.6	40.8
0.55	166.7	-41.8	37.4
0.50	151.5	-38.0	34.0
0.45	136.4	-34.2	30.6
0.40	121.2	-30.4	27.2
0.35	106.1	-26.6	23.8
0.30	90.9	-22.8	20.4
0.25	75.8	-19.0	17.0
0.20	60.6	-15.2	13.6
0.15	45.5	-11.4	10.2
0.10	30.3	-7.6	6.8
0.05	15.2	-3.8	3.4
0.00	0.0	0.0	0.0

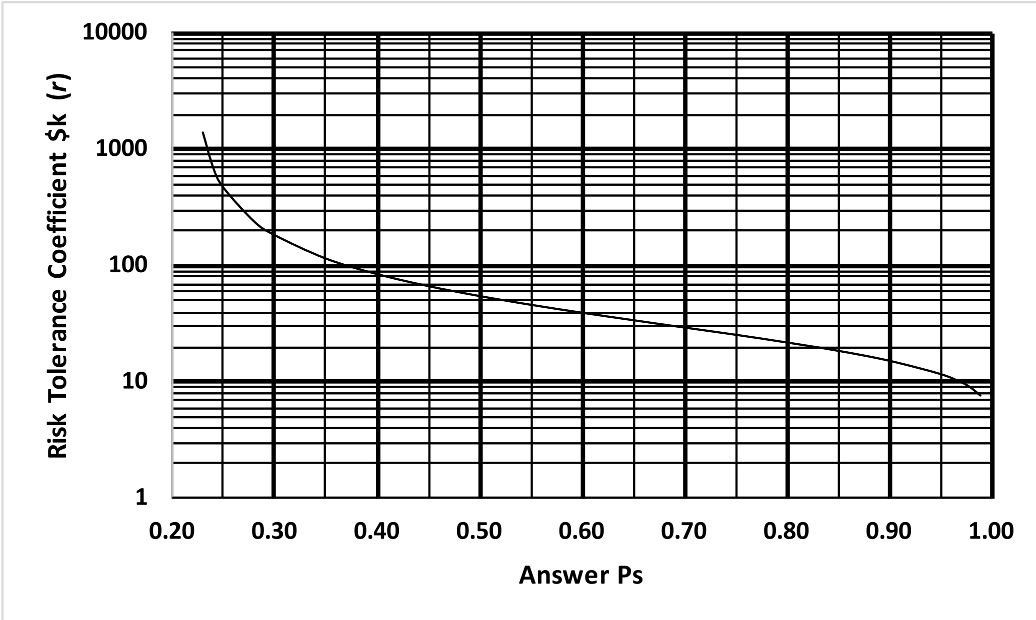
For this last question type, you are asked to consider an economic project that is too large for your personal portfolio or size company.

Answers

For your answers to the three questions, we have the imputed values of *r* (or *RTC*) for each.

You will notice that reading a precise *r* from the charts is very difficult. That is why it is so nice to have equations for the utility function. If your answer is not among values the corresponding table, then you can interpolate to get a reasonably precise *r*.

1. Ps Question

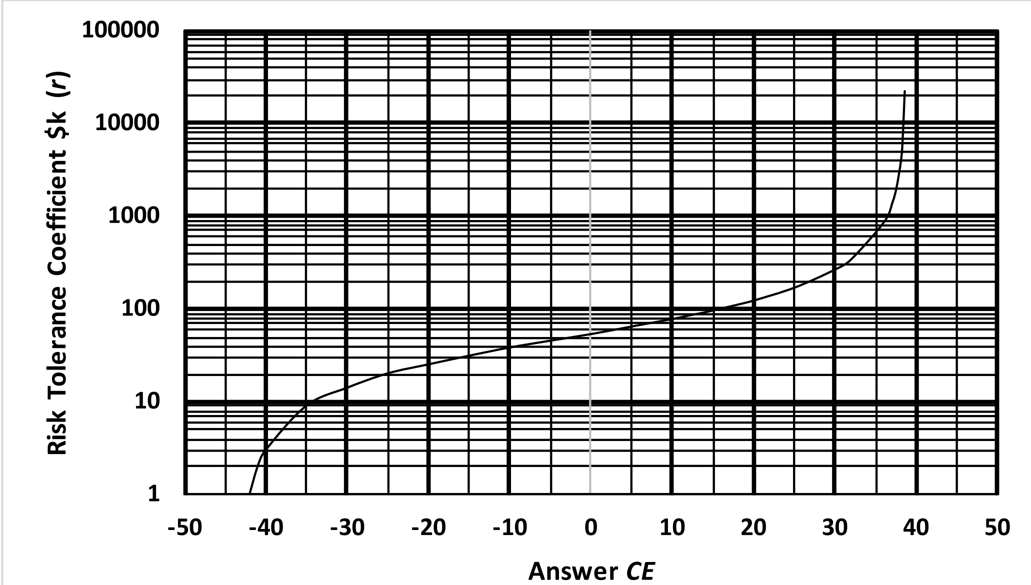


Answer example: Ps = .80 corresponds to *r* = $21.8k

These are the data used for the chart:

Answer	RTC
0.23	1397
0.24	701
0.25	470
0.28	240
0.30	183
0.35	115
0.40	84.5
0.45	66.6
0.50	54.7
0.55	46.0
0.60	39.3
0.65	33.9
0.70	29.3
0.75	25.4
0.80	21.8
0.85	18.5
0.90	15.2
0.95	11.7
0.97	10.0
0.98	9.0
0.99	7.6

2. CE Question

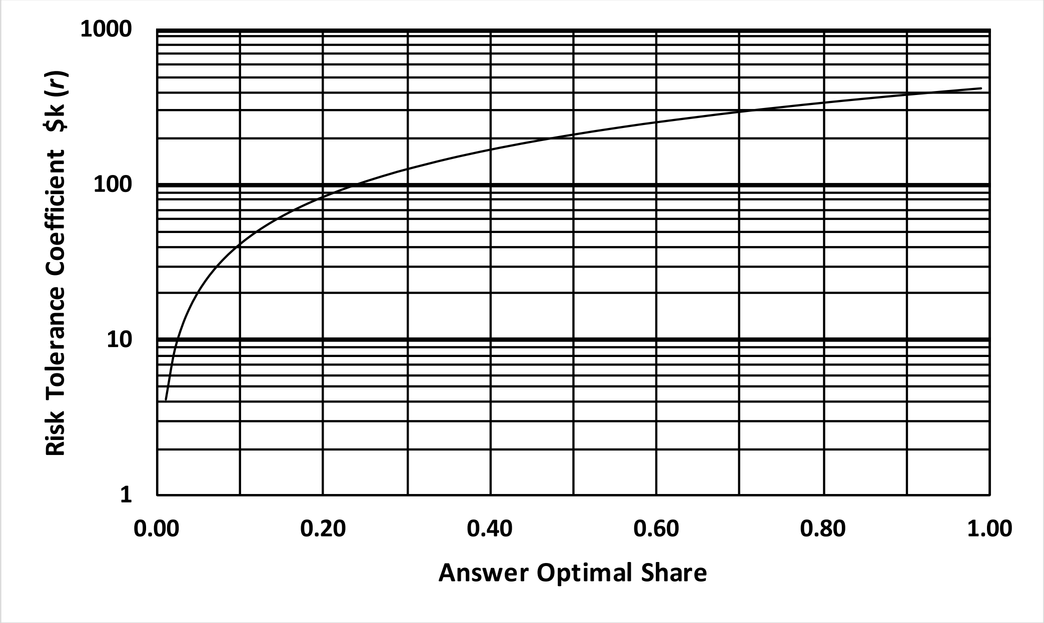


Answer Example. If *CE* = -$10k, then *r* = $38k.

Answer	RTC
38.5	21924
38.2	5549
38	3699
37.5	2024
37	1393
36	862
32	343
30	264
25	168
20	122
18	110
16	100
14	91
12	84
10	77
7	69
5	64
0.001	53
-5	45
-10	38
-15	31
-20	25
-25	20
-30	14
-35	9
-40	3
-41	2
-42	1

then

3. Share Question



Answer Example. If optimal *Share* = .10, then *r* = $42.4k.

Note: the optimization curve shown with the question was calculated for *r* = $150k.

Share	RTC
0.99	419.7
0.90	381.5
0.80	339.1
0.70	296.7
0.60	254.4
0.55	233.2
0.50	212.0
0.45	190.8
0.40	169.6
0.35	148.4
0.30	127.2
0.28	118.7
0.26	110.2
0.24	101.7
0.22	93.3
0.20	84.8
0.19	80.5

Share	RTC
0.18	76.3
0.17	72.1
0.16	67.8
0.15	63.6
0.14	59.3
0.13	55.1
0.12	50.9
0.11	46.6
0.10	42.4
0.09	38.2
0.08	33.9
0.07	29.7
0.06	25.4
0.05	21.2
0.04	17.0
0.03	12.7
0.02	8.5
0.01	4.2

Utility Calculations with Excel

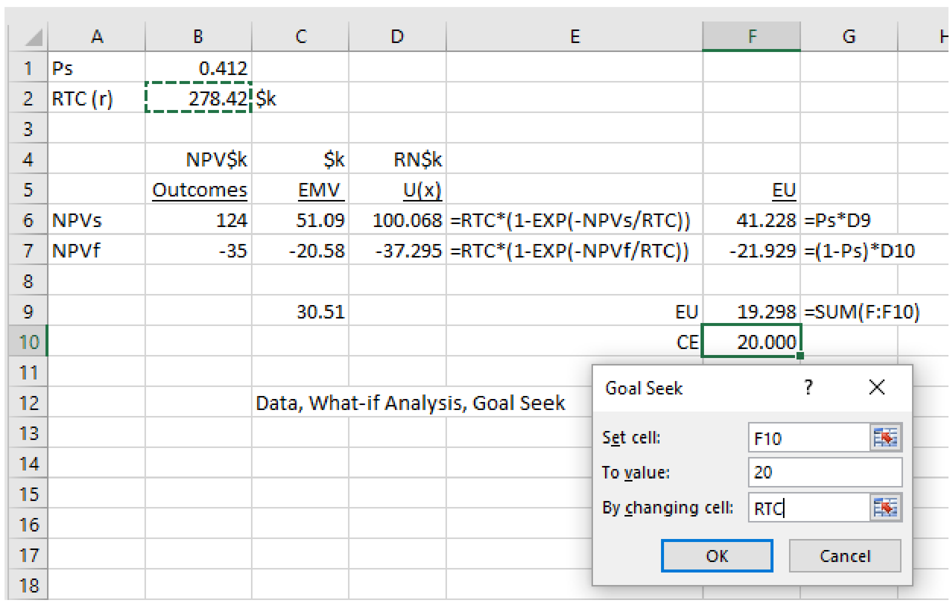
It is worthwhile to check some of these calculations. Excel is an excellent tool for experimenting with risk policy.

Solving for *r* with the Ps and CE question types are a goal-seek operations. Solving for *r* with the Optimal WI is a bit more difficult and requires optimization. Excel can solve these with its Goal Seek and Solver tools.

For example, let's solve for a Question 4, seen earlier. This is a CE type question. Set up a worksheet, as shown below, to solve for a *CE*. Put in 100 or some arbitrary starting value for *r*. Suppose your assessment is *CE* = $20k. We want the calculated *CE* in F10 to be 20, by changing *r* (in cell B2).

Solve with this pull-down menu sequence:

Data
What-if analysis
Goal Seek to set F10 (*CE*)
To Value 20 (an example answer)
By Changing (cell B2, named) RTC.



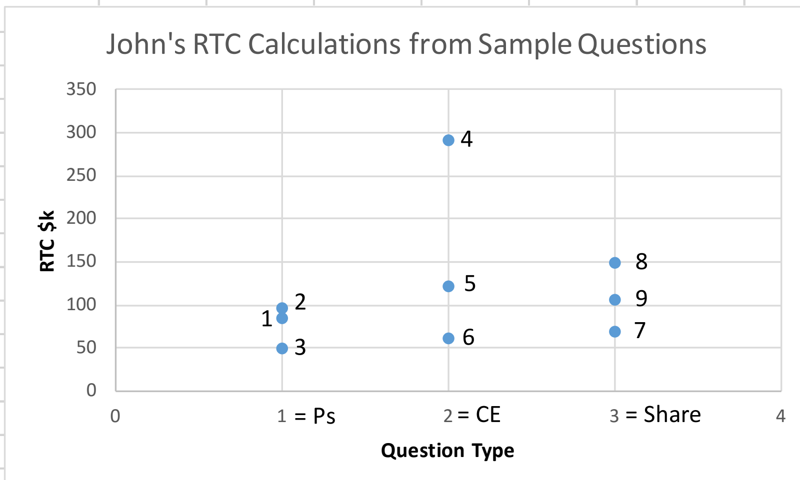
Spreadsheet solution, solving for *r* given minimum *P*s.

Practice and Feedback

UEP makes it easy to practice with many and varied questions. I suggest recording the *r*'s calculated from your answers. You may want to your question parameters, your answers, and the returned values into Excel and plot your performance.[10]

I hope that you will become "calibrated" with practice and feedback. However, that is not the point. The purpose is to give you confidence in the exponential utility function and, having seen a good many *r*'s returned, to select an *r* value of implementing as your risk policy.

Qtype	Perspect	NPVs	NPVf	Ps	Answer	RTC
1	1	124	-35	0	0.4	85
1	1	108	-15	0	0.2	96
1	1	210	-60	0	0.7	50
2	1	126	-34	0.4	20	293
2	1	93	-43	0.6	20	122
2	1	55	-13	0.75	30	63
3	1	890	-220	0.55	0.1	69
3	1	313	-76	0.376	0.35	150
3	1	511	-149	0.503	0.2	106



Example export data file (top) with a chart showing the distribution of *r* values. Clearly, more calibration practice is needed. The UEP idea is that, with practice, you will become calibrated and able to answer these questions with reasonably consistent *r* values.[11]

[10] A planned UEP enhancement will provide some means to export session data.

[11] A statistic for measuring relative dispersion is the *coefficient of variation* (*CV*). *CV* = *standard deviation* / *mean* = $s / \bar{x}$. A reasonable calibration target would be *CV* = .25. That's 39% of the *CV* for John's values shown above. Likely, your *r* distribution will be positively-skewed. Assuming a lognormal shape, an 80% confidence range for *r*'s would be the mean +33%/-29%.

We welcome questions, comments, and suggestions.

John Schuyler

john@maxvalue.com

1-303-693-0067 (U.S. Mountain time zone = GMT -6 or -7)

www.maxvalue.com

■