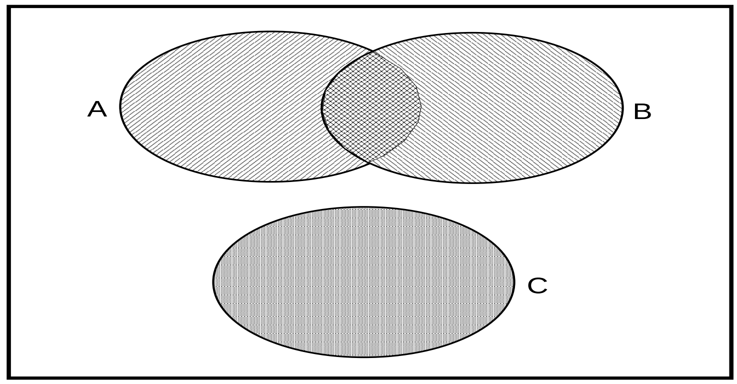
6. BASIC PROBABILITY CONCEPTS

Venn Diagrams and Boolean Algebra

A useful aid for visualizing events and probabilities is a Venn diagram:



All possible events that we relate together are included in the diagram. The shapes are not important, although the graph will be more meaningful if areas are drawn roughly proportional to the probability of the corresponding event. The areas, representing probabilities, are dimensionless. The area totals 1.

Some useful notation, in what is called logic or ***Boolean algebra***:

$\overline{A}$ means not A

P(A) means the probability of A

AB or A·B means A and B (the intersection)

P(A|B) means the probability of A, given B

A+B means A *or* B (the union)

In the scenario in the Venn diagram, above, there are five possible outcomes:

1.	$A \cdot \overline{B}$	$A \cdot \overline{B} \cdot \overline{C}$ = A only
2.	$B \cdot \overline{A}$	$B \cdot \overline{A} \cdot \overline{C}$ = B only
3.	$A \cdot B$	intersection of A and B
4.	C	$C \cdot \overline{A} \cdot \overline{B}$ = C only
5.	$\overline{A} \cdot \overline{B} \cdot \overline{C}$	neither A nor B nor C

Key Probability Theorems

$P(A) + P(\bar{A}) = 1$. This obvious concept is called the ***complement rule***.

The ***union*** of areas A and B represent is represented by A+B. The notation A+B is not exclusive; that is, it includes A·B:

$$P(A+B) = P(A) + P(B) - P(A \cdot B)$$

This is the called the ***addition rule***.

A term like $P(A \cdot B)$ is called a ***joint probability***. It is calculated as $P(A \cdot B) = P(A)\, P(B|A) = P(B)\, P(A|B)$. This is the ***multiplication rule***. In the special case where the variables are independent, this simplifies to: $P(AB) = P(A)\, P(B)$. Indeed, a test for independence is:

If $P(A|B) = P(A)$ then A and B are independent.

These three rules form the core knowledge needed to work with probabilities.

Conditional Probability

The probability of B occurring when it is known that some event A has occurred is called a conditional probability. A useful relation (read "the probability of B, given A") is:

$$P(B|A) = \frac{P(A \cdot B)}{P(A)}$$

This formula is the *multiplication rule* rearranged. Above, A and B may be dependent. Bayes' rule, described in a later section, is a generalization of this.

Binomial Distribution

The binomial Distribution can be used to represent, at least in approximation, many real-world problems. This probability distribution applies to experiments where there are only two possible outcomes, usually “success” or “failure,” for each trial. The trials are independent and, and the probability of success is the same for any trial. For a p chance of success, the probability of x successes in n trials is:

$$P(x,n,p) = C_x^n\, p^x\, (1-p)^{n-x}$$

where the "binomial coefficient" $C_x^n = \binom{n}{x} = \frac{n!}{x!\,(n-x)!}$

For example, the probability of 4 successes in 10 trials of an event with P(success)=0.3 is

$$P(4,10,0.3) = C^{10}_{4}\,(0.3)^{4}\,(1-0.3)^{10-4} = \frac{10!}{4!(10-4)!}\,(0.3)^{4}(0.7)^{6}$$

$$= \frac{3\,628\,800}{(24)(720)}\,(0.0081)\,(0.1176) = 0.2001$$

The binomial distribution can be approximated by the normal distribution when $np > 5$ and $n(1-p) > 5$. The mean $\mu = np$ and the variance $\sigma^2 = np(1-p)$.

Some Popular Probability Distribution Shapes