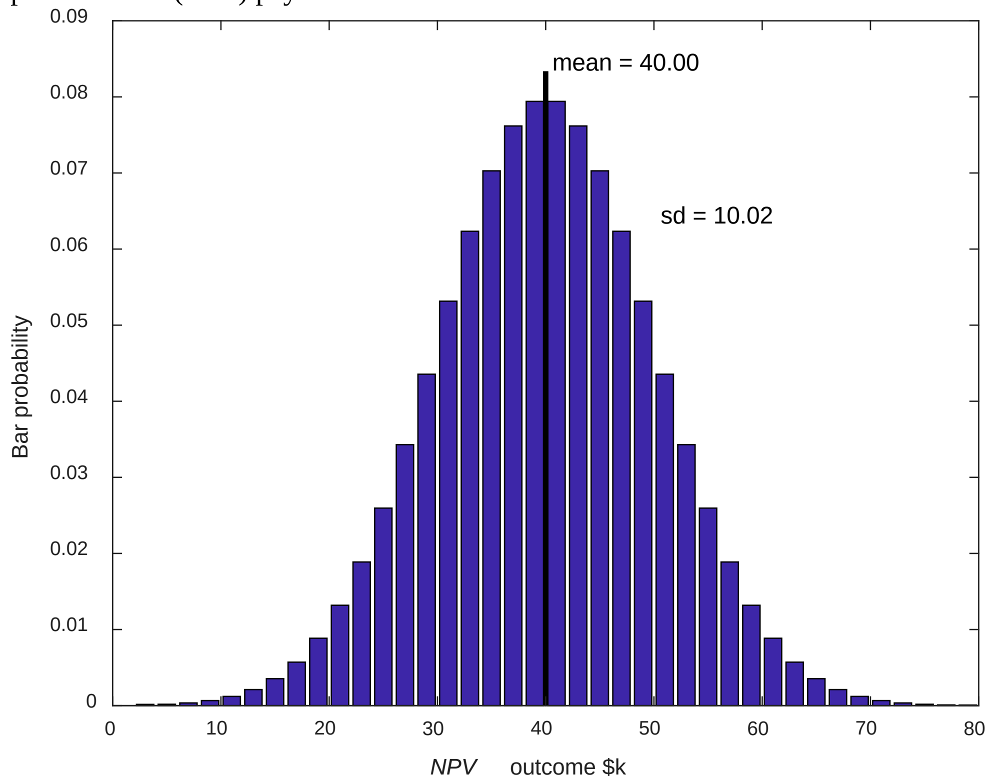
Utility Elicitation Questions I

Determining *r*'s from *CE*s of Normal Distributions

The Utility Elicitation Program (UEP) presently considers only binary risks. This document presents two examples to test whether users might like to consider continuous distributions questions. Please send your comments and suggestions to john@maxvalue.com.

Question 1. Assured positive outcome.

Consider an uncertain asset that you can purchase or already own. You accept that this distribution represents the net present value (*NPV*) payoff:



What is your *certain equivalent* (*CE*) for this distribution? Consider this from either a buy or sell perspective:

- What is the most you would be willing to pay to acquire this asset?
- Or, if you already own it, what is the smallest amount for which you would be willing to sell it?

Consider your answer for *CE* carefully. The next page lets you find the risk tolerance coefficient (*r*) corresponding to your *CE* answer.

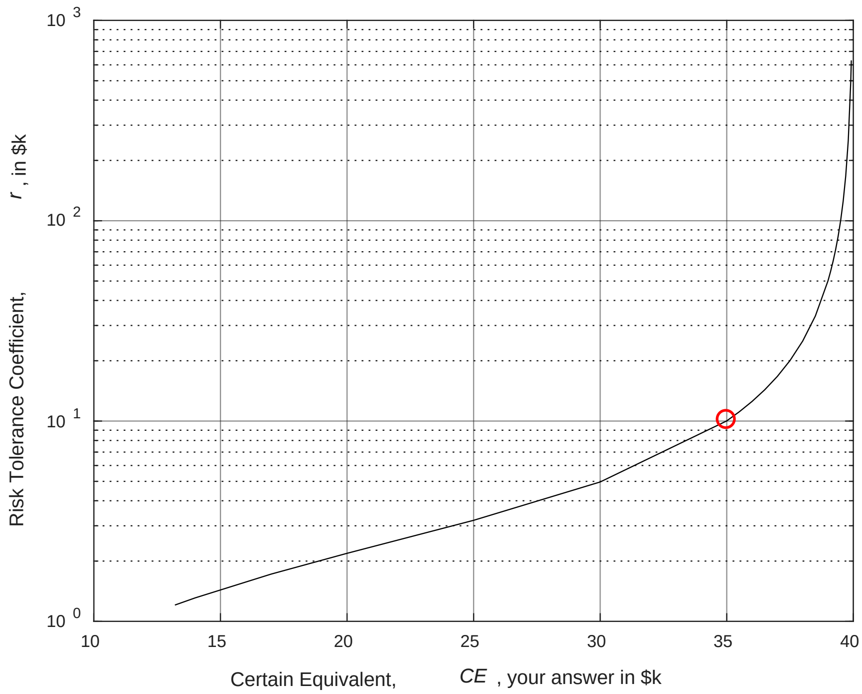
For those with a statistics background, this histogram is that of a normal distribution (a.k.a. Gaussian distribution or "bell curve") with a mean (μ) of $40k and standard deviation (σ) of $10k.

There is about 68% probability that the *NPV* outcome will be between $\mu \pm \sigma$, $30-50k.
There is about 95% probability that the *NPV* outcome will be between $\mu \pm 2\sigma$, $20-60k.
There is about 99.7% probability that the *NPV* outcome will be between $\mu \pm 3\sigma$, $10-70k.

Solution. Translate your certain equivalent (*CE*) answer to your risk tolerance coefficient (*r*) by finding or interpolating between values in the table or by using the chart. For example, if your answer is *CE* = $35k, this corresponds to *r* ≅ $10k.

Amounts in $k

CE	***r***
13.20	1.21
14.00	1.31
17.00	1.72
20.00	2.19
25.00	3.19
30.00	4.96
35.00	10.02
35.50	11.14
36.00	12.53
36.50	14.32
37.00	16.71
37.50	20.06
38.00	25.08
38.50	33.44
39.00	50.14
39.10	55.72
39.20	62.68
39.30	71.64
39.40	83.59
39.50	100.32
39.60	125.40
39.70	167.15
39.80	250.95
39.90	501.05
39.92	631.70
40.00	∞



The probability-weighted *NPV* outcome, $40k, is the *expected monetary value* (*EMV*).

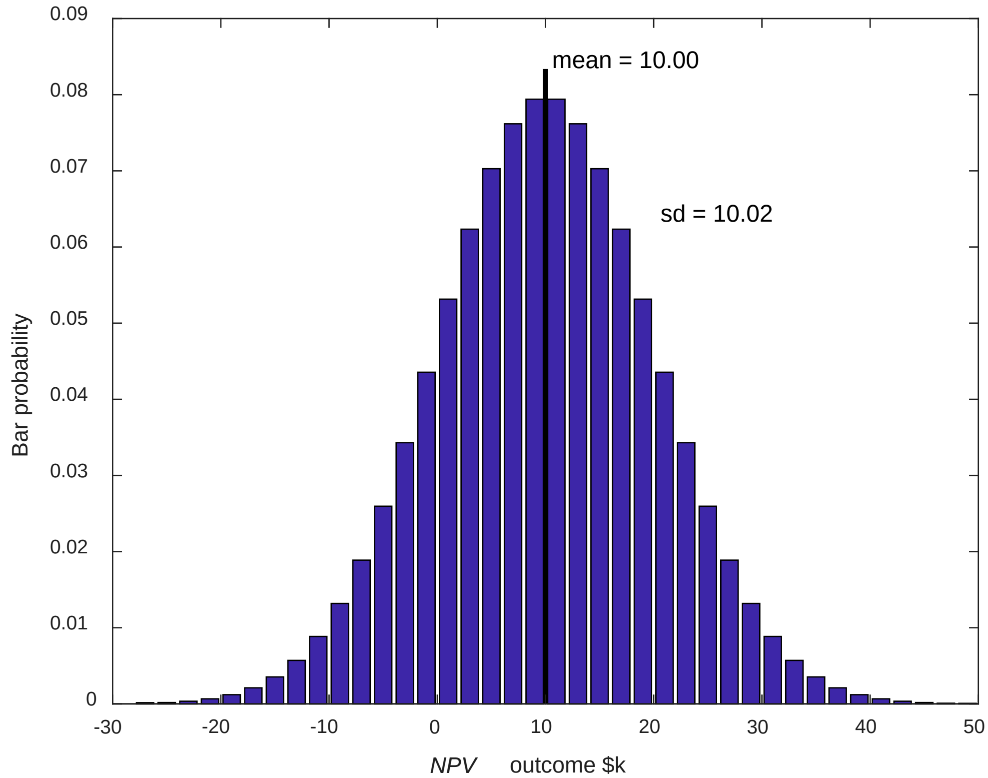
A risk-neutral person's *CE* equals the *EMV*. This person would be indifferent between having $40k cash in hand or the asset represented by the *NPV* distribution (chart on the prior page).

A risk-seeking person's *CE* would be *higher* than $40k.

And a risk-averse person (most of us) would value this at less than the *EMV*. The risk tolerance coefficient (*r*) measures your degree of risk aversion. As your *r* increases, your *CE* approaches *EMV*.

Question 2. Substantial risk of loss.

Consider an uncertain asset that you can purchase or already own. You accept that this distribution represents the net present value (*NPV*) payoff:



What is your *certain equivalent* (*CE*) for this distribution? Consider this from either a buy or sell perspective:

- What is the most you would be willing to pay to acquire this asset?
- Assume you already own it. What is the smallest amount for which you would be willing to sell it?

Consider your *CE* answer carefully. Because of the loss potential, your *CE* may even be negative; this means someone would have to *pay you* to take the project or asset.

The next page has a table and chart to convert your *CE* answer to a risk tolerance coefficient (*r*).

Supplemental information. The probability of a loss (*NPV* < $0) is .16.

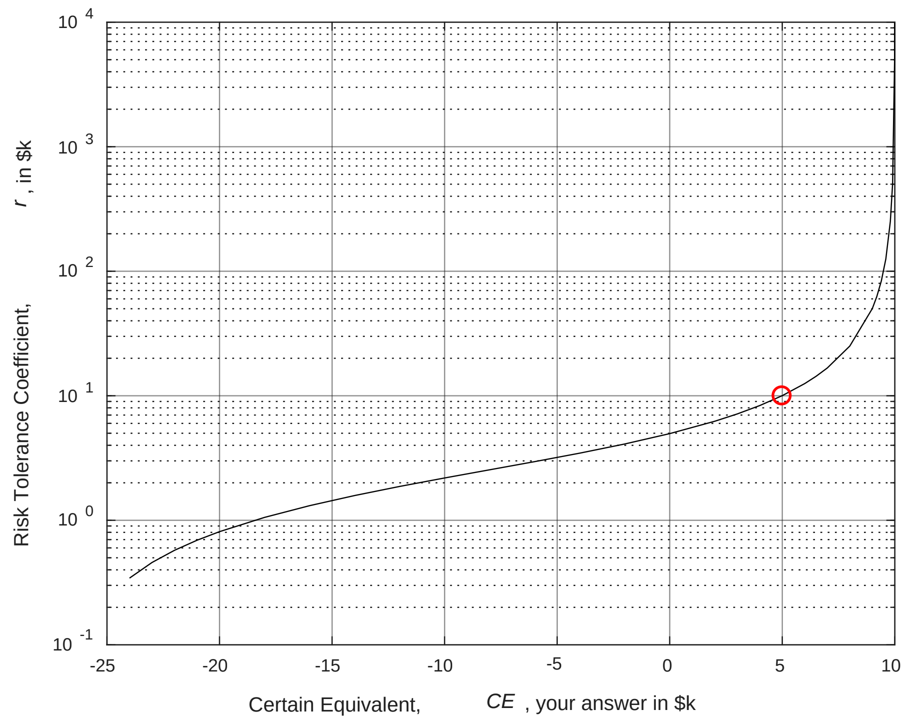
As with the prior question, this is also a normal distribution. This one has a mean (μ) of $10k and standard deviation (σ) of $10k.

There is about 68% probability that the *NPV* outcome will be between $\mu \pm \sigma$, $0-20k.
There is about 95% probability that the *NPV* outcome will be between $\mu \pm 2\sigma$, -$10-+$30k.
There is about 99.7% probability that the *NPV* outcome will be between $\mu \pm 3\sigma$,
-$20-+$40k.

Translate your certain equivalent (*CE*) answer into your risk tolerance coefficient (*r*) by finding or interpolating between values in the table or by using the chart. For example, if your answer is *CE* = \$5k, this corresponds to *r* ≅ \$10k.

Amounts in \$k

CE	***r***
-24.00	0.34
-23.00	0.46
-22.00	0.57
-21.00	0.69
-20.00	0.81
-18.00	1.05
-16.00	1.31
-14.00	1.58
-12.00	1.87
-10.00	2.19
-8.00	2.54
-6.00	2.96
-4.00	3.46
-2.00	4.10
0.00	4.96
2.00	6.24
3.00	7.14
4.00	8.34
5.00	10.02
6.00	12.53
6.50	14.32
7.00	16.71
8.00	25.08
9.00	50.14
9.20	62.68
9.40	83.59
9.60	125.40
9.80	250.95
9.90	501.05
9.99	4984.58
9.995	9897.74
10.00	∞



Note that the table and chart are nearly the same as for Question 1. Here, the *EMV* and *CE* values are \$30k less (because of the *delta property*). People feel very differently about losses and gains, and, before you become "calibrated," your *r* values may vary widely between questions.

These are intended to be individual project or asset decisions in a portfolio. If you could somehow assemble 1000 identical, independent projects like this, the per-project average *NPV* outcome would be about \$10k. Because of extraordinary diversification, *EMV*s and *CE*s are about the same (with *r* = portfolio *EMV*, the *CE* is .05% less). The probability that the average outcome exceeds \$9.27k is about .99.

It works out, happily, that if the projects are reasonably independent, optimizing individual project decisions also optimizes the portfolio.